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XIV.

On the Derivation of the Mass of Jupiter from the Motion of certain Asteroids.

BY G. W. HILL.

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THAT the discussion of the observations of certain asteroids, provided they extend over a sufficient period of time, will furnish a far more accurate value of the mass of Jupiter than can be obtained from measurements of the elongation of the satellites, or from the Jupiter perturbations of Saturn, it is the object of the present note to show. And it is to be hoped that the observers will hereafter pay particular attention to those asteroids which are best adapted for the end in question.

The magnitude of the Jupiter perturbations of an asteroid depends at once on the magnitude of the least distance of the two bodies, and the greater or less degree of approach to commensurability of the ratio of their mean motions, and also on the magnitude of the eccentricity of the asteroid's orbit.

Those asteroids which lie on the outer edge of the group, and whose mean motions are nearly double that of Jupiter, will best fulfil the two first conditions named above. For they will have inequalities of long period whose coefficients will be of the order of the first power only of the eccentricities, while all other classes of long-period inequalities are necessarily of higher orders, and hence demand longer periods in order to have their coefficients brought up to an equal magnitude.

In order to exhibit the relative value of these asteroids for the purpose in view, I have computed the terms of the lowest order in the coefficients of these inequalities of long period for all the asteroids, yet discovered, whose daily mean motion lies between the limits 550″ and 650″; and have appended herewith tables, by which the value of these terms can be readily computed for any which may hereafter be discovered between these limits.

The formulæ for computing these terms are found in the *Mécanique Céleste,* Tom. I. pp. 279–281. Here i must be put equal to 2, in the terms which involve the simple power of the eccentricities. We will employ the usual notation for the designation of the elements of orbits, and make some reductions in Laplace's formulæ for the sake of more ready computation.

If we put $\gamma = \frac{2\mu' - \mu}{\mu}$ or in Laplace's notation $\frac{2n' - n}{n}$, and recollect that we need the formulæ only for the case of an inferior perturbed by a superior planet; and moreover make

$$\gamma F^{(2)} = -H, \text{ and } \gamma G^{(2)} = J,$$

$F^{(2)}$ and $G^{(2)}$ being Laplace's symbols; we shall have

$$H = \frac{1}{1-\gamma^2}\left\{\frac{2\gamma(7-\gamma^2)}{1-\gamma}\,\alpha\, b_{\frac{1}{2}}^{(2)} + \frac{3 - 11\gamma + 3\gamma^2 - \gamma^3}{2-\gamma}\left[\alpha^2\frac{d\,b_{\frac{1}{2}}^{(2)}}{d\,\alpha} + \frac{4}{1-\gamma}\,\alpha\, b_{\frac{1}{2}}^{(2)}\right] - \gamma\,\alpha^3\frac{d^2\,b_{\frac{1}{2}}^{(2)}}{d\,\alpha^2}\right\},$$

$$J = \frac{\alpha^2}{2(1-\gamma^2)}\left\{(3 - 8\gamma + \gamma^2)\left[3\,\frac{b_{\frac{1}{2}}^{(1)}}{\alpha} + \frac{d\,b_{\frac{1}{2}}^{(1)}}{d\,\alpha} - 4\right] + 8\gamma\left[3\,\frac{b_{\frac{1}{2}}^{(1)}}{\alpha} - \frac{1}{4}\,\alpha\,\frac{d^2\,b_{\frac{1}{2}}^{(1)}}{d\,\alpha^2} - 3\right]\right\}.$$

If, in the next place, K and β are derived from the equations

$$K\cos(\beta - \pi) = H\sin\varphi - J\sin\varphi'\cos(\pi' - \pi),$$
$$K\sin(\beta - \pi) = \qquad\qquad - J\sin\varphi'\sin(\pi' - \pi),$$

the inequality in longitude we are computing is

$$\frac{m'}{\gamma^2}K\sin[L - 2L' + \beta].$$

H and J may be regarded as functions of α, and are positive between the limits corresponding to $\mu = 550''$ and $\mu = 650''$. The common logarithms of these quantities are here tabulated for every 0.001 of α between the limits above mentioned; the values of $b_{\frac{1}{2}}^{(1)}$ and $b_{\frac{1}{2}}^{(2)}$ and their differentials were obtained from Runkle's *Tables of the Coefficients of the Perturbative Function.*

α	log H.	log J.	α	log H.	log J.
0.595	0.3153369	9.871828	0.605	0.3269054	9.884214
.596	.3165277	.873131	.606	.3280187	.885370
.597	.3177113	.874420	.607	.3291236	.886511
.598	.3188875	.875695	.608	.3302199	.887636
.599	.3200561	.876956	.609	.3313075	.888745
.600	.3212173	.878202	.610	.3323864	.889836
.601	.3223707	.879434	.611	.3334562	.890910
.602	.3235163	.880652	.612	.3345169	.891967
.603	.3246540	.881855	.613	.3355683	.893007
0.604	0.3257838	9.883043	0.614	0.3366103	9.894030

a	log H.	log J.	a	log H.	log J.
0.615	0.3376427	9.895036	0.638	0.3582866	9.912409
.616	.3386652	.896022	.639	.3590269	.912874
.617	.3396777	.896990	.640	.3597519	.913310
.618	.3406801	.897939	.641	.3604614	.913718
.619	.3416723	.898869	.642	.3611550	.914097
.620	.3426539	.899780	.643	.3618323	.914446
.621	.3436248	.900671	.644	.3624928	.914764
.622	.3445848	.901542	.645	.3631366	.915051
.623	.3455337	.902392	.646	.3637632	.915306
.624	.3464714	.903221	.647	.3643722	.915528
.625	.3473975	.904028	.648	.3649632	.915717
.626	.3483119	.904814	.649	.3655358	.915871
.627	.3492144	.905578	.650	.3660897	.915991
.628	.3501047	.906320	.651	.3666246	.916076
.629	.3509827	.907040	.652	.3671400	.916125
.630	.3518480	.907736	.653	.3676354	.916136
.631	.3527005	.908408	.654	.3681103	.916108
.632	.3535399	.909056	.655	.3685644	.916040
.633	.3543659	.909679	.656	.3689972	.915933
.634	.3551782	.910277	.657	.3694082	.915785
.635	.3559767	.910850	.658	.3697969	.915595
.636	.3567612	.911396	.659	.3701628	.915362
0.637	0.3575313	9.911916	0.660	0.3705053	9.915085

The values of the elements of Jupiter's orbit for the epoch 1850.0 which we will use are

$$
\begin{aligned}
m' &= \tfrac{1}{1050}, \\
\mu' &= 299.1286, \\
\log a' &= 0.7162372, \\
\varphi' &= 2^\circ\ 45'\ 54''.55, \\
\pi' &= 11^\circ\ 55'\ 2''.
\end{aligned}
$$

The values of the corresponding elements of as many of the asteroids as lie between the limits above mentioned are contained in the following table. The longitudes of the perihelia are referred to the mean equinox of 1850.0.

	μ	log a	φ	π
Hygea	634″.3118	0.4984692	5° 44′ 56″.4	234° 58′ 40″.6
Themis	636.7634	0.4973523	6 42 52.9	139 56 11.2
Euphrosyne	633.8508	0.4988680	12 44 10.3	93 27 51.5
Doris	647.1295	0.4926769	4 23 42.9	74 10 11.3
Pales	655.6209	0.4889025	13 43 18.3	32 3 13.1
Europa	650.0877	0.4913564	5 49 14.3	101 45 37.6
Mnemosyne	632.6897	0.4992106	5 58 17.1	52 58 47.8
Erato	640.8591	0.4954961	9 46 4.3	33 55 38.0
Cybele	560.8775	0.5340920	6 54 36.4	258 11 24.3
Freia	569.0505	0.5299038	10 49 12.0	93 2 36.6
Semele	652.9848	0.4900690	11 49 36.5	28 25 39.1
Sylvia	543.5800	0.5431620	4 39 22.6	337 8 6.1
Antiope	632.3591	0.4993618	11 39 2.7	293 49 3.5

The expression of the inequalities, and the length of their periods which result from the substitution of these values of the elements in the formulæ, are

Hygea	$14676''.2 \sin [L - 2L' + 228^\circ\ 58'\ 1''.4]$,	97.96 years.
Themis	$14606.2 \sin [L - 2L' + 146\ 4\ 4.5]$,	91.72 "
Euphrosyne	$28996.5 \sin [L - 2L' + 97\ 58\ 58.4]$,	99.23 "
Doris	$5086.7 \sin [L - 2L' + 85\ 41\ 49.4]$,	72.27 "
Pales	$11639.2 \sin [L - 2L' + 33\ 36\ 12.6]$,	61.57 "
Europa	$6584.4 \sin [L - 2L' + 111\ 29\ 19.2]$,	68.14 "
Mnemosyne	$12956.0 \sin [L - 2L' + 60\ 9\ 1.9]$,	102.58 "
Erato	$13654.9 \sin [L - 2L' + 36\ 21\ 16.9]$,	82.91 "
Cybele	$13145.4 \sin [L - 2L' + 251\ 13\ 31.6]$,	94.49 "
Freia	$32243.5 \sin [L - 2L' + 98\ 15\ 25.5]$,	120.93 "
Semele	$10860.7 \sin [L - 2L' + 29\ 55\ 45.1]$,	64.54 "
Antiope	$28567.8 \sin [L - 2L' + 288\ 44\ 31.6]$,	103.57 "

These expressions can be regarded as rough approximations only to the actual values of these inequalities, since all terms of the third and higher orders with respect to the eccentricities and inclinations, and of the second and higher orders with respect to the disturbing masses, have been neglected. Yet they are sufficiently exact to show the order of magnitude of the Jupiter perturbations of the asteroids in question.

The effect of these inequalities at the time of opposition will be magnified in the proportion roughly of a to $a - 1$. Thus in the case of Freia, the determination of the mass of Jupiter will depend on the observation of an arc of 12°.7.